YIELD LAG IN THE DYNAMICS OF RIGID-PLASTIC MEDIA

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Application to rigid-plastic bodies of a model of an elastoplastic medium with yield delay [lag], proposed earlier, is discussed. Statically determinate and indeterminate systems are studied. A detailed solution of the problem of dynamic bending of a circular, freely supported plate is given.

When the concept of a rigid-plastic body is used as the basis for investigating the dynamic problems of the theory of plasticity, it is assumed that the stress distribution in the parts which remain rigid during the motion of the body cannot, in principle, be determined since this follows from the definition of a perfectly rigid body. In fact, when we consider e. g. the motion of a beam with stationary yield stresses (flow hinges), the position of these hinges is determined by the condition of the bending moments at these hinges due to the external load and inertial forces retain their maximum values, and this implies that plastic hinges cannot appear at the rigid segments. The effect of yield delay characteristic for low-C steels consists of the fact that the material can be subjected and can withstand stresses appreciably in excess of the static yield stress for a certain time, and this is the yield delay time.

A model of an elastoplastic medium with yield delay is discussed in [1 – 3], while [4] also deals with the model of a rigid-plastic body with yield delay as applied to the problem of dynamic bending of a beam.

Here the first plastic hinge appears at the cross section at which some functional reaches its maximum during the first stage of motion when the hinges have yet to form. If the beam is statically determinate, then this cross section is unique.

In the case of systems of rods or plates which are statically indeterminate, the distribution of moments in the rigid state can only be obtained by making certain assumptions about the nature of the rigid regions. For this reason the model of a rigid-plastic body with yield delay needs to be made more precise.

We shall consider the rigid-plastic body as an elastic body with an infinitely large Young's modulus. The stress distribution in such a body under the action of static and quasistatic loads can be defined uniquely, since when the forces are given, the stresses at the surface are independent of Young's modulus.

1. Confining our attention to beams and plates, we obtain a model of a rigid-plastic plate with yield delay as follows. In the rigid parts of the plate the distribution of moments is given by the solution of the dynamic problem of the theory of elasticity, in which the value of the Young's modulus has been made infinite. If the static condition of plasticity has the form

$$M_e = M_s$$

where M_e is the equivalent moment (e. g. according to the von Mises or Tresca yield condition) and the condition

$$\int_0^{t_0} \varphi(M_e)\, dt = \tau \qquad (1.1)$$

where $\varphi(M_e)$ is a known delay function and τ is a material constant, holds at some $t = t_0$ on the line γ or in the region S, then for $t > t_0$

$$M_e \leqslant M_s \qquad (1.2)$$

on γ or in S.

The conditions (1.1) and (1.2) remain valid for a beam provided that M_e denotes the absolute value of the bending moment.

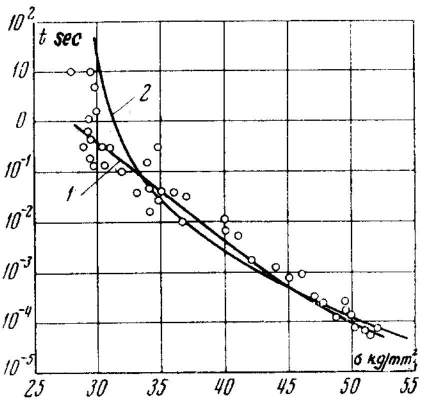

Fig. 1

Conditions (1.1) and (1.2) are constructed under the usual (for the rigid-plastic approach) assumption that the condition of plasticity for the moments has the same form as the condition of plasticity for the stresses arising in the state of plane stress. This conclusion is strictly accurate for a perfect H-beam and for a two-layer plate. In the case of a real beam or a plate the plastic region spreads across its thickness gradually, as shown in [1]; we shall not, however, consider this aspect in the present paper.

The function φ appearing under the integral sign in the delay condition (1.1) is determined experimentally. In the case of extension-compression along a single axis, it can be written in the form [5]

$$\varphi = (\sigma / \sigma^*)^\alpha \qquad (1.3)$$

where α is the material constant and σ^* denotes a certain characteristic stress.

In many problems however, the expression for φ in the form

$$\varphi = \left(\frac{\sigma - \sigma_s}{\sigma_s}\right)^n \qquad (1.4)$$

where n is the material constant and σ_s denotes the static yield point, is found more suitable.

Figure 1 depicts the experimental data of Wood and Clark [6] obtained for the time delay versus the applied load, and the curves *1* and *2* which were computed according to (1.3) and (1.4), respectively.

It may be assumed that in the case of a rigid-plastic bending of beams and plates the function φ will have the same form as that for extension-compression along a single axis. The following expression is used in the present paper

$$\varphi = \left(\frac{M - M_s}{M_s}\right)^n \quad (M > M_s), \qquad \varphi = 0 \quad (0 \leqslant M \leqslant M_s) \qquad (1.5)$$

Here M_s denotes the limiting value of the bending moment.

Equation (1.1) expresses the hypothesis of isotropic delay. In fact, it appears feasible that the release of dislocations caused by the stress acting in a single direction does not alleviate their movement in the opposite direction [7]. When applied to a beam, the above statement implies the following. If the bending moment changes its sign, then (1.1) must be computed separately for the positive and for the negative values of the moment.

Attaining the maximum value by one of these integrals determines the onset of flow. Thus a more precise form of the condition of delay for a beam is given by

$$\max\left|\int_0^t \varphi(|M|)\,dt \pm \int_0^t \varphi(|M|)\operatorname{sign} M\,dt\right| = 2\tau \qquad (1.6)$$

This condition will also be perfectly valid for a plate, with the Tresca-type criterion adopted, provided that M is understood to denote M_1, or M_2, or $M_1 - M_2$.

2. To perform the computations according to the scheme adopted, in which the rigid regions are treated as the limiting cases ($E = \infty$) of elastic regions, the distribution of moments in the rigid regions must first be determined. It will be shown that this distribution corresponds to the quasistatic solution, i.e. to the solution obtained when the inertial forces are neglected. Let us consider the dynamic bending of a plate under the load $q(x, t)$ when the boundary conditions are homogeneous. The load function $q(x, t)$ is assumed to be integrable (in quadratures) in x and continuous in t together with its first order derivative . We impose on it the following restrictions:

$$q(x, 0) = 0, \qquad |\partial q/\partial t| < m', \qquad |\partial^2 q/\partial t^2| < n' \qquad (2.1)$$

where m' and n' are arbitrarily small. We obtain the solution of the differential equation of bending

$$E\ A(w) + \rho\ d^2w/dt^2 = q \qquad (2.2)$$

as an expansion in terms of the eigenfunctions of

$$A(w) = \lambda w$$

Using the initial conditions

$$w(x, 0) = 0, \qquad (dw/dt)_{t=0} = 0$$

we obtain by the usual methods

$$w = \sum_k \tau_k u_k, \qquad \tau_k = \frac{1}{\rho\omega_k}\int_0^t \sin\omega_k (t - s)\, q_k(s)\, ds$$

where $q_k(t)$ are the Fourier coefficients of the load q. From (2.1) follows

$$q_k(0) = 0, \ |\dot{q}_k| < m, \ |\ddot{q}_k| < n \qquad (2.3)$$

where m and n are quantities proportional to m' and n' and vanishing with the latter. Integrating the expression for τ_k twice by parts and taking the initial condition into account we obtain

$$\tau_k = \frac{1}{E\lambda_k}\left\{q_k + \frac{1}{\omega_k}\left[q^{\cdot}(0)\sin\omega_k t + \int_0^t \sin\omega(t - s)\, q^{\cdot\cdot}(s)\, ds\right]\right\}$$

The expression contained within the square brackets remains bounded for any finite t. When $E \to \infty$, $\omega_k \to \infty$. The bending moments are given as products of the second order derivatives of the deflection w multiplied by E. When $E = \infty$. we have

$$E\frac{\partial^2 w}{\partial x_i \partial x_j} = \sum_k \frac{q_k}{\lambda_k}\frac{\partial^2 u}{\partial x_i \partial x_j} \qquad (2.4)$$

This corresponds to the quasistatic solution obtained by neglecting the inertial forces. Series (2.4) are guaranteed to converge at least in the mean.

The quantities m' and n' can now be made arbitrarily small. This means that constraints imposed on the mode of variation of $q(x, t)$ vanish with time. The order in which the passage to the limit is achieved is important; E tends to infinity first and is then followed by m' and n' tending to zero.

Clearly, a similar method can be applied to beams (where the uniform convergence of (2.4) can be easily shown) and to other elastic systems.

If it is assumed that q and $\partial q / \partial t$ treated as functions of t can exhibit discontinuities from the initial instant up to the time when $E \to \infty$, then a situation may arise which shall be illustrated with the case of longitudinal oscillations of a rod.

We assume that a stress σ_0 is momentarily applied to one end of the rod, the other end remaining free. As the waves reflect from the free end, the stress assumes at every cross section a value equal either to zero or to σ_0, and its mean value over a sufficiently long interval of time tends to $\sigma_0(l - x) / l$.

This is in fact the stress distribution obtained by solving the quasistatic problem when the load applied at the end is equalized by the inertial forces uniformly distributed over the whole length. If now $E \to \infty$ and consequently the velocity of propagation of the elastic waves also tends to infinity, the result given above retains its validity. The quantity $\sigma(x, t)$ however does not tend to any limit when x and t are fixed. At the same time the Fourier coefficients of $\sigma(x, t)$ tend to the Fourier coefficients of the continuous function $\sigma_0 (l - x) / l$ as E tends to infinity. For fixed x we have

$$\int_0^t \varphi(\sigma(x, t))\, dt \to \varphi(\sigma_0 (l - x)/l)\, t$$

Thus the time delay can be obtained from the quasistatic solution.

3. The method of analysis of statically indeterminate systems of beams or frames with time delay based on the theorem formulated is as follows. With the elasticity assumed, the static problem is solved for the initial state, the distribution of moments is obtained, the functional (1.6) constructed and the cross section A at which the first plastic hinge, corresponding to the time t_1 is formed, is found.

Next we consider the same system but with one plastic hinge at which the bending moment has a constant value M_s. The position of the second plastic hinge is found and the time of its formation. The process is continued until the formation of the next plastic hinge converts the system into a mechanism. In the stages that follow, the bending moments in the rigid segments are determined from the equations of motion of a system of rigid members linked by hinges. Condition (1.6) serves to determine the positions and times of appearance of successive hinges. During the analysis attention must be paid to the relative direction of rotation of the neighboring members as the formation of one hinge may lead to the disappearance of other hinges. A similar method of analysis for statically determinate systems is given in [4].

If the velocities of certain cross sections are given instead of the loads, the yield delay effect can be neglected and the conventional rigid-plastic solution remains valid.

We shall explain this using a problem investigated in [8]. A beam falls with a velocity v onto hinge supports. Thus at $t = 0$ the velocities of all cross sections of the beam except the end ones are equal to v, while the end ones are zero. The beam stops instantaneously, hence an infinitely large transverse load $q = \infty$ appears, whose impulse $qT = \rho F v$ is however finite. The bending moment M is infinite at every cross section.

Writing the condition of delay for an infinitely short time T

$$\int_0^T \left(\frac{M - M_s}{M_s}\right)^n dt \approx \left(\frac{M}{M_s}\right)^n T$$

we find that $M^n T = \infty$ when $n > 1$, from which it follows that the capacity of the material for delay will be exhausted and the value $M = M_s$ is attained at once along the whole beam, which is precisely the assumption made in the solution given in [8].

The problem of axially symmetric bending of a circular plate may be used as an example of application of the rigid-plastic analysis of a statically indeterminate system. As was said before, the possibility of plastic deformations appearing in the plate is governed by the state of stress in the rigid parts. Plastic deformations will only appear when (1.6) holds. The following scheme of the rigid-plastic solution is obtained in this case. First the static elastic problem must be solved and the distribution of moments found. At this stage a circle can be found within which the load-bearing capacity of the material will be first exhausted. The values of the bending moments will fall on this circle to their limiting values, the latter depending on the conditions of plasticity chosen. Further, the zones of plastic deformation are distributed in accordance with (1.6) in which the value of the equivalent moment appearing under the integral sign is found from the static elastic solution for the rigid part of the plate. The quantity M appearing in the expression for φ represents, in accordance with (1.5), the value of some equivalent moment corresponding to the chosen condition of plasticity.

The usual rigid-plastic analysis of bending of circular plates is based, as a rule, on the Tresca condition of plasticity. For this reason the latter condition (Fig. 2) has been assumed in the present paper in the investigation of bending with the yield lag. After determining the radial M_r and tangential M_φ moments in the rigid plate, it must be shown at which part of the $M_r \sim M_\varphi$ plane they will fall. If M_r and M_φ have the same sign (sides $A'B'$, $A'F'$, $C'D'$, $D'E'$), the equivalent moment M will correspond to this M_r or M_φ which is numerically larger. If on the other hand M_r and M_φ have opposite signs, then the value of M will correspond to the difference of these moments, which define the sides $B'C'$ or $E'F'$ of the hexagon. At the instant $t = t_0$ the bending moments fall to their limiting values corresponding to the Tresca hexagon $ABCDEF$, in which the velocities are given by the limiting values of the moments M_s. Deformation takes place in the plastic region in accordance with the rule of flow corresponding to the limiting hexagon.

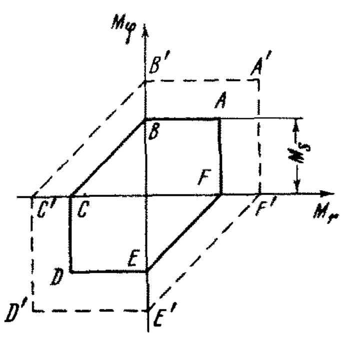

Fig. 2

4. Let us consider the bending of a circular freely supported plate acted upon by an impulsive, uniformly distributed load. A similar problem without the yield lag was studied in [9].

We show that two different forms of bending may arise, depending on the magnitude of the load. When $p_s < p < 2p_s$ (here $p_s = 6M_s / R^2$, where R is the radius of the plate, denotes the limiting load), the neutral surface initially flat assumes a conical shape. At the center of the plate the moments M_r and M_φ are equal to each other. The plate is in the state AB, its center being in the state A and the freely supported edge in the state B.

When $p > 2p_s$, a central circle appears in which we have the state A and $M_r = M_\varphi$. The radius of this circle can be found from the boundary conditions and from the condition at the boundary separating the different plastic states. The whole central circle will be in translational motion as a rigid body, while the remainder of the plate will be in the state AB and will assume the shape of a truncated cone.

On removing the load the hinge circumference separating the two regions begins to contract towards the center. After it has reached the center, the plate continues to move for some time until it comes to a complete stop.

The presence of the delay lag causes a substantial alteration to the analysis of the problem.

After the load has been applied, the plate remains stationary for some time, determined by the condition of delay (1.6), since during this time the material can withstand the stresses in excess of the limiting values.

At the time $t = t_0$ the bending moments fall to their limit static state corresponding to the Tresca hexagon $ABCDEF$. A flow takes place in the plastic region in accordance with the associated rule and the magnitude of the equivalent moment M appearing under the integral sign in (1.6) is obtained, as shown in Sect. 2, from the static elastic solution for the rigid part of the plate.

Thus, after the loads have been removed, the plate remains rigid and stationary for some time $t \leqslant t_0$. Figure 3 depicts the curves for M_r/M_s (curve $1'$) and M_φ / M_s (curve 1) during the first stage of motion. Clearly, M_φ exceeds M_r everywhere except at the point $\xi = 0$ (here $\xi = r / R$) These moments will be the most influential ones and it is with them, that the onset of the plastic deformations must be associated. (In computing the curves on Fig. 3 we assumed that the Poisson's ratio $\nu = 0.25$, the applied load $p / p_s = 4$, and denoted the moments referring to the rigid part of the plate by the subscript 1). The load capacity of the material is first exhausted at the center where the bending moment is largest. A decrease in the value of the moments then takes place, from the upper yield point determined by the applied load, to the lower yield point determined by the limiting stress. The values of the moments at the center $\xi = 0$ correspond to the point A on the yield hexagon. The plastic region then begins to expand gradually towards the plate edge, with the velocity determined by the length of delay at each cross section ξ, i.e. by the quantity $t(\xi)$. At the same time the point on the yield hexagon begins to move from A towards B. Since on the side AB we have $\varkappa = d^3w / dr^2dt = 0$ (w denotes deflection) while on the boundary separating the plastic and rigid regions the relations $\lambda = -1/r, d^2w / dr\, dt = 0$, $w = 0$ and $w' = 0$ must hold, we find that $w = w = w = 0$ everywhere in the plastic region. This means that although the center of the plate becomes plastic, no motion is observed at this stage of loading. Here $M_\varphi = M_s$, and $0 \leqslant M_r \leqslant M_s$. Integrating the equations of motion at this stage yields the following expression for M_r:

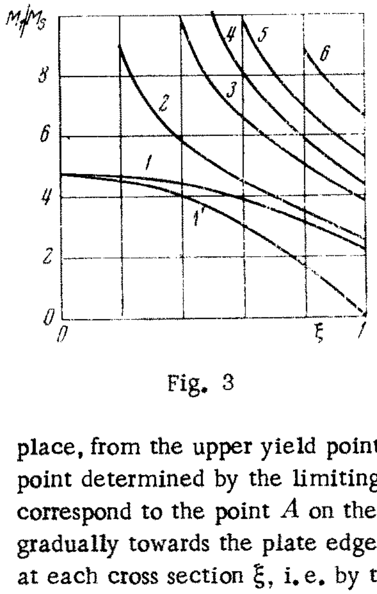

Fig. 3

$$M_{r2} = M_s(1 - k\xi^2) \quad (k = p / p_s)$$

The subscript 2 accompanying M_r indicates that the moment pertains to the region AB, and we see from the above expression that AB cannot embrace the whole plate. When it reaches the circumference denoted by ξ^* at which M_{r2} becomes equal to zero ($\xi^* = \sqrt{1 / k}$), the character of the motion must change. Thus the plate remains stationary until the region of plasticity reaches $\xi = \xi^*$.

Figure 3 gives the graphs for the moments $M_{\varphi 1}(\xi) / M_s$ in the case when the load applied is four times as large as the limiting load, for various boundaries of propagation of the plastic region (curves $2 - 4$ correspond to the values of $\rho = 0.2, 0.4, 0.5$ respectively).

When the plastic zone reaches the circumference whose coordinate is ξ^*, another plastic region begins to spread from the center of the plate and the stress in this region corresponds to the point A of the yield hexagon, i.e. $M_r = M_\varphi = M_s$. This marks the third stage of motion. Let us denote by ρ_0 the boundary between the two plastic regions. For the values $0 \leqslant \xi \leqslant \rho_0$ of the radius we have $M_r = M_\varphi = M_s$. Equations of equilibrium imply at once that the shearing force $Q = 0$ and $w^{\cdot\cdot} = 0$. For the region $\rho_0 \leqslant \xi \leqslant \rho$ we have $M_\varphi = M_s$ and $0 \leqslant M_r \leqslant M_s$ and, as was shown before, in this region $w = w^{\cdot} = w^{\cdot\cdot} = 0$. When integrating the equations of motion, we must take into account the fact that

$$M_r|_{\xi=\rho} = 0, M_r|_{\xi=\rho_0} = M_s, Q|_{\xi=\rho_0} = 0$$

Two of these conditions yield two constants of integration, while the third one gives the relation connecting ρ and ρ_0. We obtain

$$\rho / k - \rho^3 + 3 \rho\rho_0^2 - 2 \rho_0^3 = 0$$

This cubic equation expresses the relationship between the boundaries of propagation of the two plastic regions.

In the central part of the plate $(0 \leqslant \xi \leqslant \rho_0)$ the radial and tangential bending moments are equal to each other, and to the critical moment. The equations of dynamic equilibrium imply at once that the deflections in this region are under constant acceleration and are different from zero

$$w_3^{\cdot\cdot} = p / m$$

(the subscript 3 means that the accompanied quantity belongs to the region A). Integrating this expression and employing the boundary conditions $w|_{\xi=\rho_0} = 0$ and $dw/dt|_{\xi=\rho_0} = 0$, we obtain

$$w_3 = \frac{p}{2m} [t(\rho_0) - t(\xi)]^2 \tag{4.1}$$

In the region $\rho_0 \leqslant \xi \leqslant \rho$ (plastic state corresponds to the side AB of the yield hexagon) and in the rigid part $\rho \leqslant \xi \leqslant 1$ the deflections are identically zero.

Figure 3 depicts the curves for the bending moments $M_{\varphi 1}$ in the rigid part of the plate at this stage of motion, curves 5 and 6 corresponding to the values $\rho = 0.6$ and 0.8.

Motion of the boundary between the regid and the plastic region, i.e. the function $\rho(t)$ is determined by the condition of delay. Here it must be remembered that the time of delay at each point ξ is determined by the previous history of the state of stress at this point. In accordance with (1.5) and (1.6) we obtain the following equation:

$$\int_0^{t_0} \left[\frac{M_{\varphi 1}(\xi, 0)}{M_s} - 1\right]^n dt + \int_{t_0}^{t} \left[\frac{M_{\varphi 1}(\xi, \rho)}{M_s} - 1\right]^n dt = \left[\frac{M_{\varphi 1}(0, 0)}{M_s} - 1\right]^n t_0 \tag{4.2}$$

Here t_0 denotes the time of delay for the central point $\xi = 0$ of the plate at which the bending moment is equal to $M_{\varphi 1}(0,0)$. The first integral describes the contribution of $M_{\varphi 1}$ corresponding to the stage of motion during which the whole plate is rigid and plastic regions are absent. The integrand function depends only on the magnitude of the applied load and the latter determines both, the value of $M_{\varphi 1}$ for some circumference with coordinate ξ, and the time of delay t_0 at the center where $\xi = 0$. The second integral can be transformed as follows. Changing the variable of integration from t to $\rho\,(dt = t_\rho\, d\rho)$ we obtain

$$\int_{t_0}^{t}\left[\frac{M_{\varphi 1}(\xi,\rho)}{M_s}-1\right]^n dt = \int_0^{\xi}\left[\frac{M_{\varphi 1}(\xi,\rho)}{M_s}-1\right]^n t'_\rho d\rho$$

Equation (4.2) now transforms into the integral Volterra equation of the first kind

$$\frac{1}{t_0}\int_0^{\xi}\left[\frac{M_{\varphi 1}(\xi,\rho)}{M_s}-1\right]^n t'_\rho d\rho = \left[\frac{M_{\varphi 1}(0,0)}{M_s}-1\right]^n - \left[\frac{M_{\varphi 1}(\xi,0)}{M_s}-1\right]^n \quad (4.3)$$

We shall first show that $t_\rho = 0$ at the center $\xi = 0$. To do this we must assume that $\rho \to 0$ and $\xi \to 0$ in the expression for the moments $M_{\varphi 1}(\xi, \rho)$. Expanding the terms entering (4.2) in small values of ξ and assuming that $t' = a + b\xi^\alpha$, we easily obtain $a = 0$ and $\alpha = 1$. Thus $t' = b\xi$ at small ξ, i.e. if $\xi = 0$ we also have $t' = 0$.

The integral equation (4.3) can be easily solved on a digital computer for any material characteristics, i.e. for any values of n. However, to illustrate the analysis performed above we shall use the following approximate method which presents no great difficulties when n have integral values (we shall use $n = 1$ for simplicity). We approximate the functions $M_{\varphi 1}(\xi, \rho)$ writing polynomials in ρ for each ξ and represent t' also in the form of the following polynomials

$$t' = a_1\rho + a_2\rho^2 + \ldots + a_s\rho^s = \sum_{i=1}^{s} a_i\rho^i \quad (4.4)$$

We now insert (4.4) into (4.3) and assume that it holds for $\xi = \xi_i$ $(i = 1, 2, \ldots, s)$. This yields a system of linear algebraic equations defining the coefficients a_i. Graphical integration of $t'(\xi)$ yields the function $t(\xi)$ itself, and the latter is shown in Fig. 4. (The values of $n = 1$ and $s = 5$ were used for the purpose of integration). We see that for the load chosen ($k = 4$) the time in which the plastic region expands from the center to the plate edge is very short and equal to only $0.47\ t_0$. At the instant $t_2 = 1.47\ t_0$ the whole plate becomes plastic, the state A in which the deflections are given by (4.1) prevailing in the region $0 \leqslant \xi \leqslant \rho_0^* = 0.67$ and state ABin which the deflections are identically zero, in the remaining part of the plate. Figure 4 also shows the form of deflections in the plate at the instant when the plastic region reaches the freely supported edge and the whole plate becomes plastic, the nondimensional quantity $W = wm / pt_0^2$ plotted on the ordinate axis. After the whole plate has become plastic, its motion becomes identical to that without the yield lag. Only the initial conditions of the problems are different.

In the present paper we consider the case when the load applied to the plate acts upon it only for a short period and is then removed. If we find that the duration of the application of the load affects the influence of the yield lag on the magnitude of the residual deflections in a varying manner. If the duration t_3 of the impulse is so short that the plastic deformation had not yet time to appear (capacity of the material for delay is not exhausted at any point $0 \leqslant t_3 < t_0$) or if the plate is in that stage of loading during which the plastic region AB is expanding but the deflections are still identically zero ($t_0 \leqslant t_3 < t_1 = 1.07\ t_0$), then upon the removal of load the plate reverts to the rigid state at once and the moments become equal to zero everywhere.

When the load is removed at the instant t_3 such that two zones, plastic and rigid are present ($1.07\ t_0 \leqslant t_3 < 1.47\ t_0$) then we have the case of bending of a plate with deflections and rates of deflections given at some part of the plate ($0 \leqslant \xi \leqslant \rho_0$) at the initial instant. This case however is unimportant from the practical point of view, as the time of propagation of the plastic zone is very short ($0.40 t_0$) and it is fairly difficult to remove the load precisely during this interval.

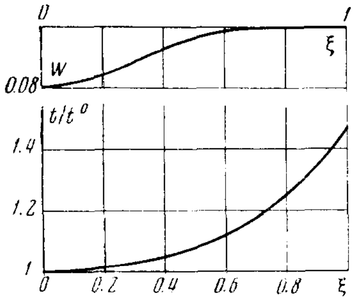

Fig. 4

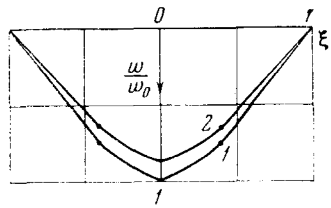

Fig. 5

The case when the removal of load takes place when the whole plate is in the plastic state appears to be the most important one. The analysis of its motion follows that of the problem without the yield lag. Figure 5 shows the form of the final deflection in the problems with (curve 2) and without (curve 1) the yield lag. (Here the duration of the load is $t_3 = 20\ t_0$). In the problem without the yield lag the total time of motion up to the complete stop (with $t_3 = 20 t_0$) is $T = 80 t_0$, while with yield lag taken into account it becomes $T = 74.1 t_0$.

From Fig. 5 we can see that taking yield lag into account leads to a decrease in the residual deflections. This decrease is associated with the first two stages of motion characterized by the complete absence of any deflections. In the first of these stages the carrying capacity of the plate is nowhere exhausted and the whole plate is still in the rigid state, while the second stage corresponds to the propagation of the plastic region AB and is also characterized by the absence of deflections. The form of the plate itself will also be different in these two problems. In the problem without yield lag the meridian of the plate is curved when $0 \leqslant \xi \leqslant \rho^*$ where ρ^* is the coordinate at which a stationary hinge circle is formed in the loaded plate. By [9] this coordinate is given by the condition

$$(1 - \rho^*)^2 (1 + \rho^*) = 2 / k$$

When $\xi > \rho^*$, the meridian is rectilinear and its inclination to the point $\xi = 0$ is not zero. Moreover it becomes discontinuous at $\xi = \rho^*$. In the case with yield lag the meridian is curved when $0 \leqslant \xi \leqslant \rho_0^*$ ($\rho_0^* = \rho_0(t_2)$ where $t_2 = 1.47 t_0$ and corresponds to the instant at which the plastic region reaches the plate edge) and there is no discontinuity in the slope at the point $\xi = \rho_0^*$. A discontinuity will however appear at the point $\xi = \rho^*$ corresponding to the previous case.

The deflections developed by the time during which the plastic region has spread over the whole plate and reached its edge are extremely small, therefore the discrepancy in the form of the meridian becomes significant only when the times of action of the impulse become comparable with the time of delay at the plastic center $\xi = 0$.

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